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## **Transonic Nonlinear Aeroelastic Simulations Using An Harmonic Balance Method**

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# Transonic Nonlinear Aeroelastic Simulations Using An Harmonic Balance Method

6<sup>th</sup> European CFD conference - ECCOMAS 2014

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School of Mechanical and Aerospace Engineering  
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# Outline

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- 1 Overview
- 2 Motivation
- 3 Aeroelastic-Harmonic Balance
- 4 Validation
- 5 Results
- 6 Conclusions

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# Motivation

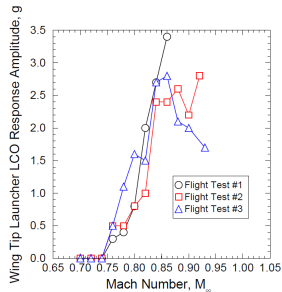


Figure : F-16 Flight Test Variability<sup>1</sup>

<sup>1</sup>Thomas *et al.* , AIAA paper 2010-2632

# Motivation Simulation Requirements

Hence, for such problems we need:

- Physics Based (nonlinear) Simulations → High-Fidelity Analysis
- Uncertainty & Variability → Probabilistic/Possibilistic
- Large Parameter Spaces → Efficient Methods
- Integration of Available Measurements → Model Updating and Calibration

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# Harmonic Balance Methodology

- Consider a generic structural system described by the equation of motion:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}$$
$$\dot{\mathbf{W}} + \mathbf{R} = 0$$



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$$\dot{\mathbf{W}} + \mathbf{R} = 0$$

- Assuming the solution is periodic, it is approximated by a truncated Fourier series of  $N_H$  harmonics and fundamental frequency  $\omega$

$$x(t) \approx \hat{x}_0 + \sum_{n=1}^{N_H} (\hat{x}_{2n-1} \cos(n\omega t) + \hat{x}_{2n} \sin(n\omega t))$$

# HB for CFD

Consider the general form of the fluid Euler equations:

$$\frac{\partial \mathbf{W}(t)}{\partial t} + \mathbf{R}(t) = 0$$

---

<sup>2</sup>Hall *et al.* , “Computation of Unsteady Nonlinear Flows in Cascades Using a Harmonic Balance Technique,” AIAA J., 40(5)

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Apply Fourier expansion to  $\mathbf{W}$  &  $\mathbf{R}$  and re-write eq. in the frequency domain:

$$\omega \mathbf{A} \hat{\mathbf{W}} + \hat{\mathbf{R}} = 0$$

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$$\omega \mathbf{A} \hat{\mathbf{W}} + \hat{\mathbf{R}} = 0$$

Recast the Fourier coefficients in the time domain and solve:<sup>2</sup>

$$\underbrace{\frac{\partial \tilde{\mathbf{W}}}{\partial \tau}}_{\text{pseudo-time}} + \underbrace{\omega \mathbf{D} \tilde{\mathbf{W}}}_{\text{Unsteady Source}} + \underbrace{\tilde{\mathbf{R}}}_{\text{residual}} = 0$$

---

<sup>2</sup>Hall *et al.* , "Computation of Unsteady Nonlinear Flows in Cascades Using a Harmonic Balance Technique," AIAA J., 40(5)

# HB for CFD

where:

$$\mathbf{D} = \mathbf{E}^{-1} \mathbf{A} \mathbf{E}, \quad \mathbf{E} = \begin{bmatrix} 1/2 & 1/2 & \cdots & 1/2 \\ \cos t_0 & \cos t_1 & \cdots & \cos t_{2N_H} \\ \sin t_0 & \sin t_1 & \cdots & \sin t_{2N_H} \\ \cos 2t_0 & \cos 2t_1 & \cdots & \cos 2t_{2N_H} \\ \sin 2t_0 & \sin 2t_1 & \cdots & \sin 2t_{2N_H} \\ \vdots & \vdots & \cdots & \vdots \\ \cos N_H t_0 & \cos N_H t_1 & \cdots & \cos N_H t_{2N_H} \\ \sin N_H t_0 & \sin N_H t_1 & \cdots & \sin N_H t_{2N_H} \end{bmatrix}; \quad \tilde{\mathbf{W}} = \begin{bmatrix} \mathbf{W}(t_0) \\ \mathbf{W}(t_1) \\ \vdots \\ \mathbf{W}(t_{2N_H}) \end{bmatrix}$$

$$t_i = \frac{i2\pi}{2N_H + 1}, \quad i = \{0, 1, 2, \dots, 2N_H\}$$

# Aeroelastic - Harmonic Balance (A-HB)

Consider a generic, undamped, structural system described by the equation of motion:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}(\mathbf{W})$$

# Aeroelastic - Harmonic Balance (A-HB)

Consider a generic, undamped, structural system described by the equation of motion:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}(\mathbf{W})$$

Recast in state-space form

$$\dot{\mathbf{y}} = \mathbf{A}_s \mathbf{y} + \mathbf{B}_s \mathbf{F}$$

where:

$$\mathbf{A}_s = \begin{bmatrix} 0 & \mathbf{I} \\ \mathbf{M}^{-1}\mathbf{K} & 0 \end{bmatrix}; \quad \mathbf{B}_s = \begin{bmatrix} 0 \\ \mathbf{M}^{-1} \end{bmatrix}; \quad \mathbf{y} = \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix}$$

# HB for CFD

Applying the same transformations, we obtain a coupled system in the HB format:

$$\frac{\partial \tilde{\mathbf{y}}}{\partial \tau} + \omega \mathbf{D}_s \tilde{\mathbf{y}} + (\mathbf{A}_s \tilde{\mathbf{y}} + \mathbf{B}_s \mathbf{F}) = 0$$

$$\frac{\partial \tilde{\mathbf{W}}}{\partial \tau} + \omega \mathbf{D}_f \tilde{\mathbf{W}} + \tilde{\mathbf{R}} = 0$$



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$$\frac{\partial \tilde{\mathbf{W}}}{\partial \tau} + \omega \mathbf{D}_f \tilde{\mathbf{W}} + \tilde{\mathbf{R}} = 0$$

Even CFD unsteady problems can be computed efficiently, as long as  $N_H$  does not become excessive and  $\omega$  is known.

# LCO Prediction

- However, we don't know  $\omega_{LCO}$

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- ▶ However, we don't know  $\omega_{LCO}$  🙄
- ▶ New approach based on frequency updating

# LCO Prediction HB with Frequency Updating

Define the following residual:

$$\mathbf{R}(\omega, \mathbf{y}) = \omega \mathbf{D}_s \mathbf{y} - (\mathbf{A}_s \mathbf{y} + \mathbf{B}_s \mathbf{f})$$

From the residual, define the following figure of merit:

$$\mathbf{L} = \frac{1}{2} \mathbf{R}^T \mathbf{R} = \frac{1}{2} [\omega \mathbf{D}_s \mathbf{y} - (\mathbf{A}_s \mathbf{y} + \mathbf{B}_s \mathbf{f})]^T [\omega \mathbf{D}_s \mathbf{y} - (\mathbf{A}_s \mathbf{y} + \mathbf{B}_s \mathbf{f})]$$

# LCO Prediction HB with Frequency Updating

We then propose the following algorithm to determine LCOs:

Find  $[\omega, \mathbf{F}]$  that minimizes  $\mathbf{R}(\omega, \mathbf{y})$  using:

$$\frac{\partial \tilde{\mathbf{y}}}{\partial \tau} + \omega \mathbf{D}_s \tilde{\mathbf{y}} + (\mathbf{A}_s \tilde{\mathbf{y}} + \mathbf{B}_s \mathbf{F}) = 0$$



$$\frac{\partial \tilde{\mathbf{W}}}{\partial \tau} + \omega \mathbf{D}_f \tilde{\mathbf{W}} + \tilde{\mathbf{R}} = 0$$



$$\frac{\partial \mathbf{L}_n}{\partial \omega} = \left( \mathbf{D}_y - \mathbf{B}_s \frac{\partial \mathbf{f}}{\partial \omega} \right)^T [\omega \mathbf{D}_y - (\mathbf{A}_s \mathbf{y} + \mathbf{B}_s \mathbf{f})]$$

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# Forced Motion Predictions

## AGARD CT-6

Case	$M_\infty$	$\alpha_m$	$\alpha_0$	$k$	$x_m$
CT6	0.796	0	1.01	0.202	0.25

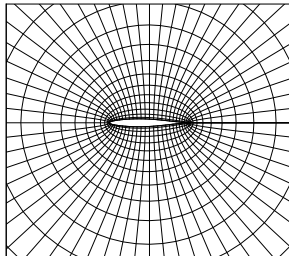
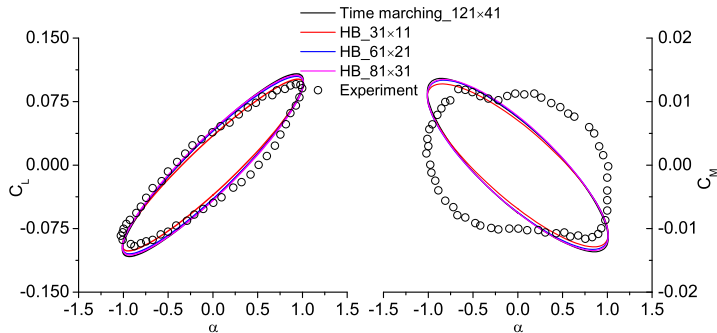


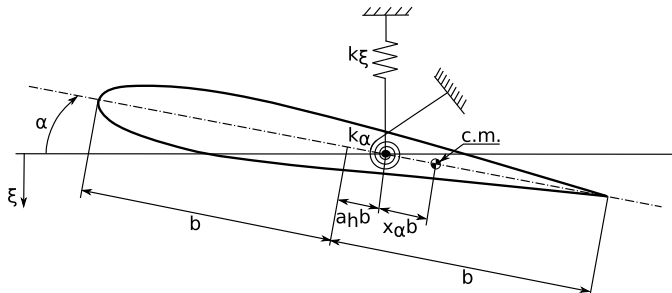
Figure : NACA 64A010



# Forced Motion Predictions



# Forced Motion Predictions



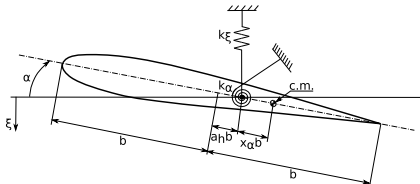
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## Pitch - Plunge Aerofoil

Problem widely studied in the community<sup>3</sup>:

$$\begin{aligned} m\ddot{h} + S_\alpha\ddot{\alpha} + K_h &= -q_\infty c C_l \\ S_\alpha\ddot{h} + I_\alpha\ddot{\alpha} + K_\alpha &= q_\infty c^2 C_m \end{aligned}$$



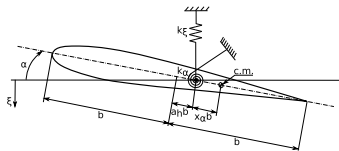
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Problem widely studied in the community:

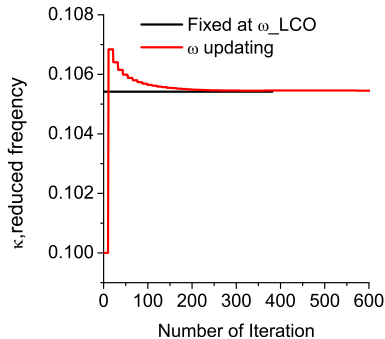
$$\mathbf{M} = \begin{bmatrix} 1 & x_\alpha \\ x_\alpha & r_\alpha^2 \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} \left(\frac{\omega_h}{\omega_\alpha}\right)^2 & 0 \\ 0 & r_\alpha^2 \end{bmatrix}$$

$$\mathbf{f} = \begin{bmatrix} -C_l \\ 2C_m \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \frac{h}{b} \\ \alpha \end{bmatrix}, \quad V = \frac{U_\infty}{\sqrt{\mu\omega_\alpha b}}$$

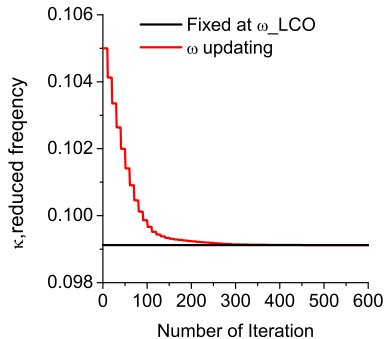


# Pitch - Plunge Aerofoil Solver Convergence

## Frequency Convergence



(a)  $V_s = 0.725$

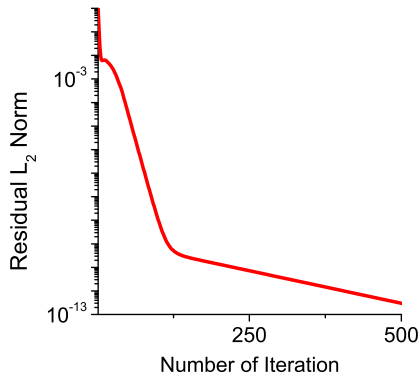
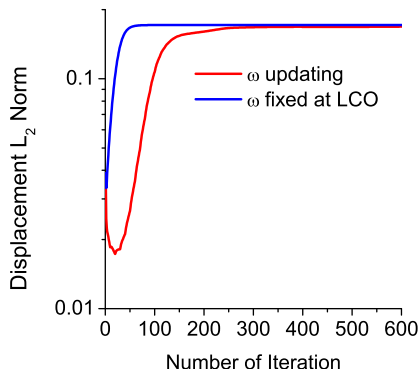


(b)  $V_s = 0.80$

A-HB initialised at flutter conditions

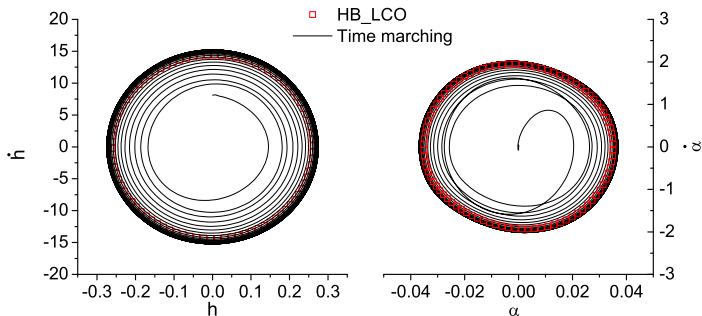
# Pitch - Plunge Aerofoil Solver Convergence

Frequency Convergence ( $V_s = 0.80$ )



# Pitch - Plunge Aerofoil Solver Convergence

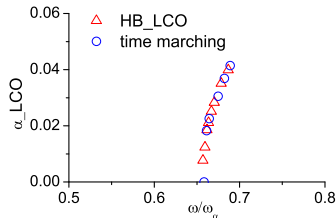
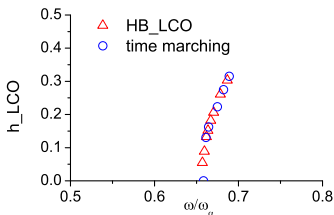
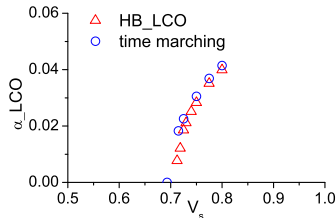
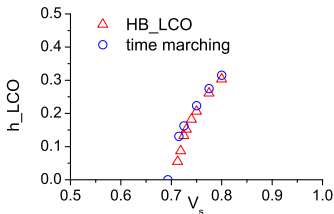
Position-Velocity Diagram ( $V_s = 0.80$ )





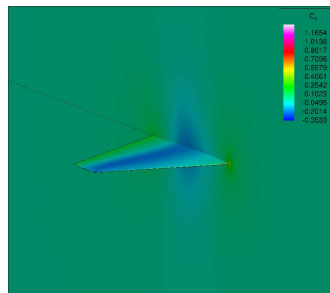
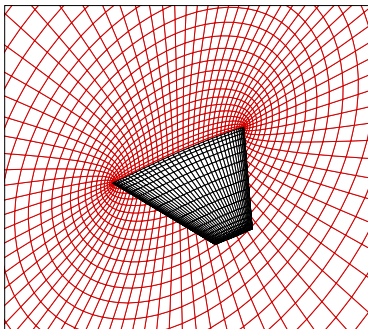
# Pitch - Plunge Aerofoil Solver Convergence

## LCO Amplitude



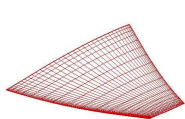
# Delta Wing

## Generic Fighter Delta Wing

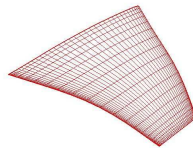


# Delta Wing

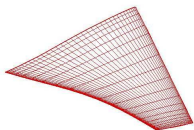
## Structural Model based on 2D shell elements



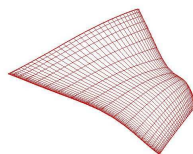
3.94Hz



12.88Hz



15.82



27.56Hz

# Delta Wing

LCO Prediction -  $M_\infty = 0.91$ ;  $\alpha = 0^\circ$ ;  $q_{flutter} = 0.759q_{sl}$

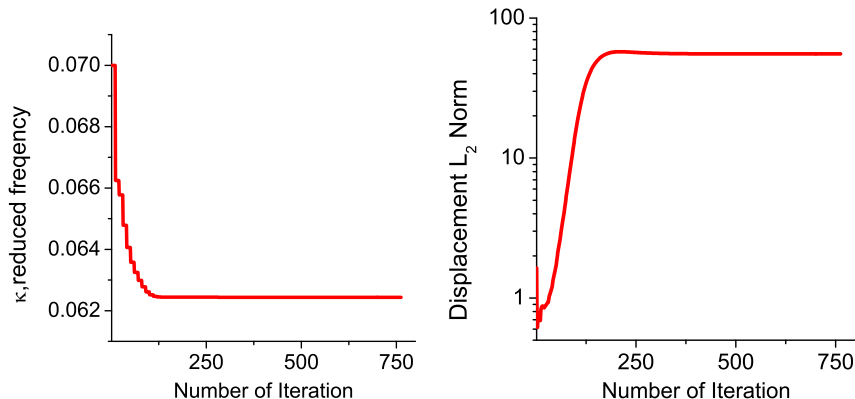
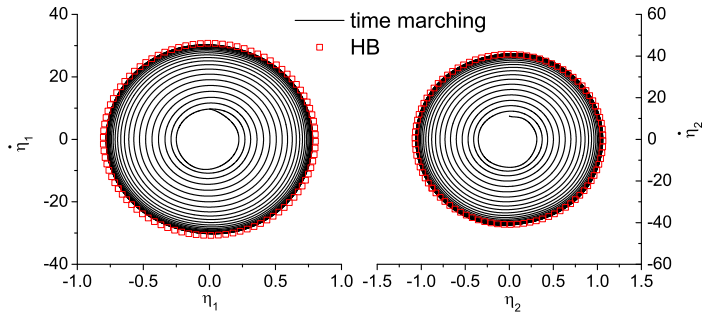


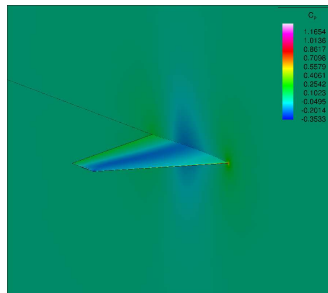
Figure :  $q_{LCO} = 0.85q_{sl}$

# Delta Wing

Position-Velocity Diagram ( $q_{LCO} = 0.85q_{sl}$ )



# Delta Wing



# Solver Efficiency

	Aerofoil		Delta Wing	
	Wall Clock [s]	Speed-Up	Wall Clock [h]	Speed-Up
Time Marching	3990	1.0	204	1.0
1 Harmonic	282	14.1	8	25.5
2 Harmonics	480	8.3		
3 Harmonics	1050	3.8		
4 Harmonics	1224	3.3		

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# Conclusions

- a new formulation for LCO prediction based on a Harmonic Balance method has been presented.

# Conclusions

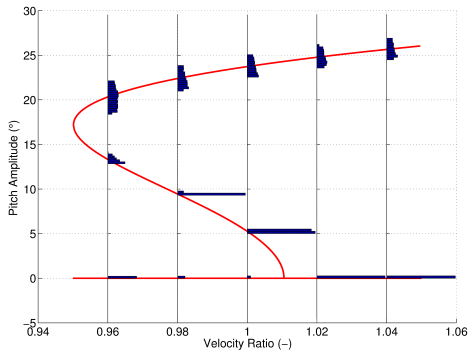
- a new formulation for LCO prediction based on a Harmonic Balance method has been presented.
- LCO conditions are determined using a frequency updating algorithm that accounts for both aerodynamic and structural influences

# Conclusions

- a new formulation for LCO prediction based on a Harmonic Balance method has been presented.
- LCO conditions are determined using a frequency updating algorithm that accounts for both aerodynamic and structural influences
- the new approach shows significant speed-ups w.r.t. time-domain, without compromising accuracy

# Future Work

- Include stronger nonlinearities.
- Quantify uncertainties

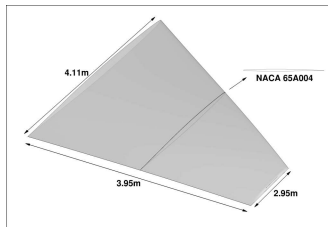
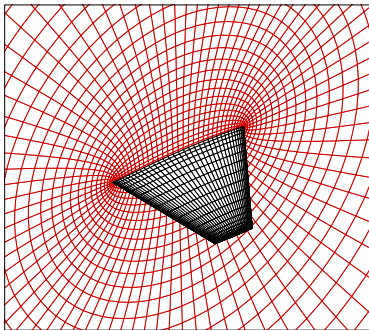


# Q & A

Thank you for your attention

Questions Welcome

# Q & A



- Grid size: 160k
- NACA 65A004

## Q &amp; A

## Flutter Calculations:

## Eigenvalue Problem

$$\begin{bmatrix} A_{ff} & A_{fs} \\ A_{sf} & A_{ss} \end{bmatrix} \mathbf{p} = \lambda \mathbf{p}$$

this can be reduced to a small nonlinear eigenvalue problem, by applying the Schur Complement Method<sup>4</sup>

## Schur Formulation

$$S(\lambda) \mathbf{p}_s = \lambda \mathbf{p}_s$$

where

$$S(\lambda) = (A_{ss} - \lambda I) - A_{sf}(A_{ff} - \lambda I)^{-1}A_{fs}$$

<sup>4</sup>Badcock, Timme, Marques, *et al.* Envelope searches and uncertainty analysis with transonic aeroelastic simulation. Progress in Aerospace Sciences, 47(5), 2011.

## Q &amp; A

## Flutter Calculations:

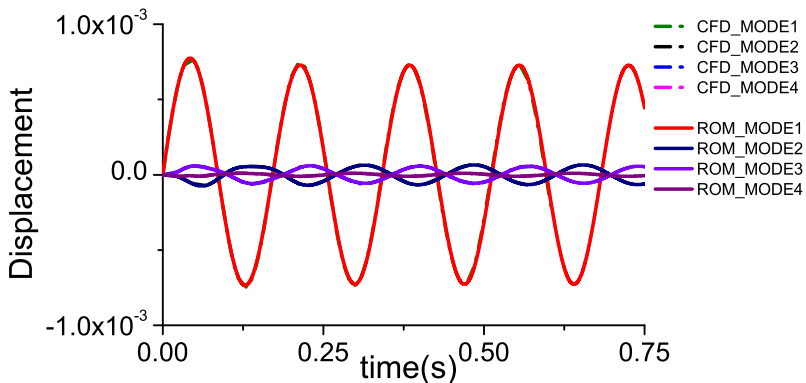


Figure : Delta Wing Flutter response with Volterra Series 808<sup>th</sup> ROM